

# PENPOL SCHOOL 

## Mathematics Policy

Agreed: April 2022
Review Date: April 2023

## Aims

At Penpol School, we believe that mathematics equips children with an essential set of skills to enable them to succeed in their education and fulfil their potential in later life. Mathematics is vital for making sense of the world around us and as a means of solving a wide range of problems. As such, we aim to engender an enthusiasm for the subject and an understanding of the importance it holds.

We believe it is essential that children become highly numerate and develop an awareness of their mathematical understanding, which they are able to communicate, reflect upon and evaluate in order to further their own learning. This nurtures independent and confident learners who show persistence in working towards their learning goals and have high expectations of themselves across all subjects, including mathematics.

## Curriculum Provision

At Penpol School, we use White Rose planning and the White Rose calculation policy from EYFS through to Year 6. This unified approach allows for the progression of skills across the school to be easily mapped and understood. As we practice a mastery approach to mathematics, there is an expectation that children move through the content at broadly the same speed although there is a recognition that some children may need extra support, either within the classroom or in smaller intervention groups and that some children may need to work individually to explore concepts that provide extra challenge.

## Teaching and planning

Teachers are expected to use regular, recorded assessment for learning as a means of ensuring that their teaching is responsive to the individual needs of all children. Daily plans, as provided by White Rose, allow for a range of approaches including practical activities using manipulatives, pictorial and the abstract. We take the approach that the use of manipulatives provides the first essential building blocks and a springboard to the pictorial and the abstract and but can continue to be used in all year groups to support learning. All children are exposed to varied fluency on a daily basis to help embed their learning across a range of different contexts.

The golden thread of oracy that runs through the school, means that all children are exposed to daily reasoning activities in which they are encouraged to make sense of their mathematics through discussion and explanation. This golden thread is also visible in the UCR (Understanding, Communicating and Reflecting) approach to reasoning and problem-solving using CUBES (Circle, Underline, Box up, Explain, Solve).

## Assessment

Daily formative assessment is at the heart of maths teaching at Penpol Primary School. A whole class marking policy is used which enables teachers to make informed decisions on the direction of lessons as well as planning for direct daily intervention to ensure that no child is left behind. Assessment of each unit is carried out once the unit is complete, allowing teachers to plan for intervention and to cover any class-wide gaps in learning.

## Professional development

At Penpol School, we are fully committed to the continued professional development of our staff; as such we are an active member of the Cornwall and Devon Maths Hub which has provided and continues to provide support and training for teachers across all year groups. In addition, an active open doors policy allows teachers to support each other through a collaborative learning approach.

## Year 1-6

Calculation Policy
Addition and Subtraction

## \#MathsEveryoneCan

## Notes and Guidance

## Calculation Policy

Welcome to the White Rose Maths Calculation Policy.
This document is broken down into addition and subtraction, and multiplication and division.

At the start of each policy, there is an overview of the different models and images that can support the teaching of different concepts. These provide explanations of the benefits of using the models and show the links between different operations.


Each operation is then broken down into skills and each skill has a dedicated page showing the different models and images that could be used to effectively teach that concept.

There is an overview of skills linked to year groups to support consistency through out school. A glossary of terms is provided at the end of the calculation policy to support understanding of the key language used to teach the four operations.


## Part-Whole Model



$$
\begin{array}{ll}
7=4+3 & 7-3=4 \\
7=3+4 & 7-4=3
\end{array}
$$



## Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

## Bar Model (single)

## Concrete



Discrete


Combination


## Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

## Bar Model (multiple)

## Discrete



$$
7+3=10
$$



## Continuous


$7-3=4$
$2,394-1,014=1,380$

## Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

## Number Shapes



$6+4$


7+3


8+2


9+1

## Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1 , they can see that the other number decreases by 1 to find all the possible number bonds for a number.

## Cubes



$$
7=4+3
$$

## Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.
$7=3+4$

$7-3=4$

$7-3=4$
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

## Ten Frames (within 10)



$$
\begin{aligned}
& 4+3=7 \\
& 3+4=7 \\
& 7-3=4 \\
& 7-4=3
\end{aligned}
$$

## 4 is a part.

 3 is a part. 7 is the whole.$\qquad$


$$
4+3=7
$$


$7-3=4$

## Benefits

When adding and subtracting within 10 , the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

## Ten Frames (within 20)



## Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

## Bead Strings

## -00-00000000--000-0000000-

 $-00-000000000000000000-$$-000-00000000000000000-$


## Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10 .
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2+8=10$, move one bead, $3+7=10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20 .

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

## Number Tracks

$5+3=8$


$$
10-4=6
$$



$$
8+7=15
$$



## Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

## Number Lines (labelled)



## Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

## Number Lines (blank)

$35+37=72$

$35+37=72$

$72-35=37$


## Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

## Straws

## $7+6=13$


$42-17=25$

bundle together groups of 10


## Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

## Base 10/Dienes (addition)



38
$+23$
61
1


## Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children. How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? ( 10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column) How many ones do we have left? (Write in ones column) Repeat for each column.

## Base 10/Dienes (subtraction)



## Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

## Place Value Counters (addition)


(1)
.

## Place Value Counters (Subtraction)



## Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

## Addition

| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Add two 1-digit <br> numbers to 10 | 1 | Part-whole model <br> Bar model <br> Number shapes | Ten frames (within 10) <br> Bead strings (10) <br> Number tracks |
| Add 1 and 2-digit <br> numbers to 20 | 1 | Part-whole model <br> Bar model <br> Number shapes <br> Ten frames (within 20) | Bead strings (20) <br> Number tracks <br> Number lines (labelled) <br> Straws |
| Add three 1-digit <br> numbers | 2 | Part-whole model <br> Bar model | Ten frames (within 20) <br> Number shapes |
| Add 1 and 2-digit <br> numbers to 100 | 2 | Part-whole model <br> Bar model <br> Number lines (labelled) | Number lines (blank) |
| Straws |  |  |  |
| Hundred square |  |  |  |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Add two 2-digit numbers | 2 | Part-whole model <br> Bar model <br> Number lines (blank) Straws | Base 10 <br> Place value counters Column addition |
| Add with up to 3-digits | 3 | Part-whole model Bar model | Base 10 <br> Place value counters Column addition |
| Add with up to 4-digits | 4 | Part-whole model Bar model | Base 10 <br> Place value counters Column addition |
| Add with more than 4 digits | 5 | Part-whole model Bar model | Place value counters Column addition |
| Add with up to 3 decimal places | 5 | Part-whole model Bar model | Place value counters Column addition |

Skill: Add 1-digit numbers within 10 Year: 1









## Subtraction

| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Subtract two 1-digit <br> numbers to 10 | 1 | Part-whole model <br> Bar model <br> Number shapes | Ten frames (within 10) <br> Bead strings (10) <br> Number tracks |
| Subtract 1 and 2-digit <br> numbers to 20 | 1 | Part-whole model <br> Bar model <br> Number shapes <br> Ten frames (within 20) | Bead string (20) <br> Number tracks <br> Number lines (labelled) <br> Straws |
| Subtract 1 and 2-digit <br> numbers to 100 | 2 | Part-whole model <br> Bar model <br> Number lines (labelled) | Number lines (blank) <br> Straws <br> Hundred square |
| Subtract two 2-digit <br> numbers | 2 | Part-whole model <br> Bar model <br> Number lines (blank) <br> Straws | Base 10 <br> Place value counters <br> Column addition |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Subtract with up to 3- <br> digits | 3 | Part-whole model <br> Bar model | Base 10 <br> Place value counters <br> Column addition |
| Subtract with up to 4- <br> digits | 4 | Part-whole model <br> Bar model | Base 10 |
| Subtract with more than <br> 4 digits <br> Column addition |  |  |  |
| Subtract with up to 3 <br> decimal places | 5 | Part-whole model <br> Bar model | Place value counters <br> Column addition |









## Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.
Complement - in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference - the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange - Change a number or expression for another of an equal value.

Minuend - A quantity or number from which another is subtracted.

Partitioning - Splitting a number into its component parts.

Reduction - Subtraction as take away.
Subitise - Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.
Total - The aggregate or the sum found by addition.

## Year 1-6

## Calculation Policy Multiplication and Division

## \#MathsEveryoneCan

## Notes and Guidance

## Calculation Policy

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## 

## Bar Model



Girls


## Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

## Number Shapes


$5 \times 4=20$
$4 \times 5=20$

## $8: 8998888$

$$
18 \div 3=6
$$

## Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd $\times$ odd $=$ even, odd $\times$ even $=$ odd, even $\times$ even $=$ even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18 .

## Bead Strings

## $-000-000-000-000-000-$

$$
\begin{aligned}
& 5 \times 3=15 \\
& 3 \times 5=15
\end{aligned} \quad 15 \div 3=5
$$

## -00000-00000-00000-

$$
\begin{aligned}
& 5 \times 3=15 \\
& 3 \times 5=15
\end{aligned} \quad 15 \div 5=3
$$

$-0000-0000-0000-0000-000-$
$4 \times 5=20$
$20 \div 4=5$
$5 \times 4=20$

## Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 - Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

## Number Tracks



$$
\begin{aligned}
& 6 \times 3=18 \\
& 3 \times 6=18
\end{aligned}
$$


$18 \div 3=6$

## Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0 . Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

## Number Lines (labelled)


$4 \times 5=20$
$5 \times 4=20$

$20 \div 4=5$

## Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0 .
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

## Number Lines (blank)



## Benefits

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

## Base 10/Dienes (multiplication)



## Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2 -digits by 2 -digits.

## Base 10/Dienes (division)

## Benefits



| Tens | Ones |
| :---: | :---: |
| 1 |  |
| 1 |  |

$$
72 \div 3=24
$$



Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the partwhole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

## Place Value Counters (multiplication)



## Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2digit numbers by 2 -digit numbers.

## Place Value Counters (division)



1223
44892

## Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

## Times Tables

| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Recall and use multiplication and division facts for the 2-times table | 2 | Bar model Number shapes Counters Money | Ten frames <br> Bead strings <br> Number lines <br> Everyday objects |
| Recall and use multiplication and division facts for the 5-times table | 2 | Bar model Number shapes Counters Money | Ten frames <br> Bead strings Number lines Everyday objects |
| Recall and use multiplication and division facts for the 10-times table | 2 | Hundred square Number shapes Counters Money | Ten frames Bead strings Number lines Base 10 |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Recall and use <br> multiplication and <br> division facts for the <br> 3-times table | 3 | Hundred square <br> Number shapes <br> Counters | Bead strings <br> Number lines <br> Everyday objects |
| Recall and use <br> multiplication and <br> division facts for the <br> 4-times table | 3 | Hundred square <br> Number shapes <br> Counters | Bead strings <br> Number lines <br> Everyday objects |
| Recall and use <br> multiplication and <br> division facts for the <br> 8-times table | 3 | Hundred square <br> Number shapes | Bead strings <br> Number tracks <br> Everyday objects |
| Recall and use <br> multiplication and <br> division facts for the <br> 6-times table | 4 | Hundred square <br> Number shapes | Bead strings <br> Number tracks <br> Everyday objects |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Recall and use <br> multiplication and <br> division facts for the <br> 7-times table | 4 | Hundred square <br> Number shapes | Bead strings <br> Number lines |
| Recall and use <br> multiplication and <br> division facts for the <br> 9-times table | 4 | Hundred square <br> Number shapes | Bead strings <br> Number lines |
| Recall and use <br> multiplication and <br> division facts for the <br> 11-times table | 4 | Hundred square <br> Base 10 | Place value counters |
| Recall and use <br> multiplication and <br> division facts for the <br> 12-times table | 4 | Nundred square lines <br> Base 10 | Place value counters |
| Number lines |  |  |  |


$-\infty-\infty-\infty-\infty-\infty-00-00-00-$


Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.

Use different models to develop fluency.


Skill: 10 times table Year: 2

## 




Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digitsthe ones are always 0 , and the tens increase by 1 ten each time.



## Skill: 8 times table

Year: 3


8

| 8 | 16 | 24 | 32 | 40 |
| :---: | :---: | :---: | :---: | :---: |
| 48 | 56 | 64 | 72 | 80 |




Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

## Skill: 6 times table




| 6 | 12 | 18 | 24 | 30 |
| :---: | :---: | :---: | :---: | :---: |
| 36 | 42 | 48 | 54 | 60 |
| 66 | 72 | 78 | 84 | 90 |

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

| Skill: 9 times table |  |  |  |  |  |  |  | Year: $\mathbf{4}$ |
| :---: | :---: | :---: | :---: | :---: | :--- | :--- | :---: | :---: |
| 9 | 18 | 27 | 36 | 45 |  |  |  |  |
| 54 | 63 | 72 | 81 | 90 |  |  |  |  |



Skill: 11 times table
Year: 4

| 11 | 22 | 33 | 44 | 55 | 66 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 77 | 88 | 99 | 110 | 121 | 132 |



| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 65 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100


## Multiplication

| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Solve one-step <br> problems with <br> multiplication | $1 / 2$ | Bar model <br> Number shapes <br> Counters | Ten frames <br> Bead strings <br> Number lines |
| Multiply 2-digit by 1- <br> digit numbers | $3 / 4$ | Place value counters <br> Base 10 | Short written method <br> Expanded written method |
| Multiply 3-digit by 1- <br> digit numbers | 4 | Place value counters <br> Base 10 | Short written method |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Multiply 2-digit by 2- <br> digit numbers | 5 | Place value counters <br> Base 10 | Short written method <br> Grid method |
| Multiply 2-digit by 3- <br> digit numbers | 5 | Place value counters | Short written method <br> Grid method |
| Multiply 2-digit by 4- <br> digit numbers | $5 / 6$ | Formal written method |  |





| Skill: Multiply 4-digit numbers by 1-digit numbers |  |  |  |  | Year: 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | When multiplying 4digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. <br> If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method. |



## Skill: Multiply 3-digit numbers by 2-digit numbers

Year: 5


| Th | H | T | O |
| :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 |
| $\times$ |  | 3 | 2 |
|  | 4 | 6 | 8 |
| 7 | 10 | 2 | 0 |
| 7 | 4 | 8 | 8 |

Children can continue to use the area model when multiplying 3digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.
$234 \times 32=7,488$

| $\times$ | 200 | 30 | 4 |
| :---: | :---: | :---: | :---: |
| 30 | 6,000 | 900 | 120 |
| 2 | 400 | 60 | 8 |


| Skill: Multiply 4-digit numbers by 2-digit numbers |  |  |  |  | Year: 5/6 <br> When multiplying 4 digits by 2-digits, children should be confident in the written method. <br> If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method. <br> Consider where exchanged digits are placed and make sure this is consistent. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| TTh | Th | H | T | 0 |  |
|  | 2 | 7 | 3 | 9 |  |
| $\times$ |  |  | 2 | 8 |  |
| $2^{2}$ | $5^{1}$ | $3^{9}$ | $7^{1}$ | 2 |  |
| $1^{5}$ | 4 | $1^{7}$ | 8 | 0 |  |
| 7 | 6 | 6 | 9 | 2 |  |
|  |  | 1 |  |  |  |
| 2,739 $\times 28=76,6$ |  |  |  |  |  |

## Division

| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Solve one-step <br> problems with division <br> (sharing) | $1 / 2$ | Bar model <br> Real life objects | Arrays <br> Counters |
| Solve one-step <br> problems with division <br> (grouping) | $1 / 2$ | Real life objects <br> Number shapes <br> Bead strings <br> Ten frames | Number lines <br> Arrays <br> Counters |
| Divide 2-digits by 1- <br> digit (no exchange <br> sharing) | 3 | Straws <br> Base 10 <br> Bar model | Place value counters |
| Divide 2-digits by 1- <br> digit (sharing with <br> exchange) | 3 | Straws <br> Base 10 | Place value counters |
| Bar model | Part-whole model |  |  |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Divide 2-digits by 1- <br> digit (sharing with <br> remainders) | $3 / 4$ | Straws <br> Base 10 <br> Bar model | Place value counters <br> Part-whole model |
| Divide 2-digits by 1- <br> digit (grouping) | $4 / 5$ | Place value counters <br> Counters | Place value grid <br> Written short division |
| Divide 3-digits by 1- <br> digit (sharing with <br> exchange) | 4 | Base 10 <br> Bar model | Place value counters |
| Part-whole model |  |  |  |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Divide 4-digits by 1- <br> digit (grouping) | 5 | Place value counters <br> Counters | Place value grid <br> Written short division |
| Divide multi-digits by <br> 2-digits (short <br> division) | 6 | Written short division | List of multiples |
| Divide multi-digits by <br> 2-digits (long division) | 6 | Written long division | List of multiples |


| Skill: Solve 1-step problems using multiplication (sharing) | Year: $1 / 2$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| There are 20 apples altogether. <br> They are shared equally between 5 bags. <br> How many apples are in each bag? | lhildren solve <br> problems by sharing <br> amounts into equal <br> groups. <br> In Year 1, children use <br> concrete and pictorial <br> representations to <br> solve problems. They <br> are not expected to <br> record division <br> formally. |
| In Year 2, children are |  |
| introduced to the |  |
| division symbol. |  |



| Skill: Divide 2-digits by 1-digit (sharing with no exchange) |  | Year: $1 / 2$ |
| :--- | :--- | :--- |
| Tens | When dividing larger <br> numbers, children can <br> use manipulatives <br> that allow them to <br> partition into tens and <br> ones. <br> Straws, Base 10 and <br> place value counters <br> can all be used to <br> share numbers into <br> equal groups. |  |
| Part-whole models |  |  |
| can provide children |  |  |
| with a clear written |  |  |
| method that matches |  |  |
| the concrete |  |  |
| representation. |  |  |


| Skill: Divide 2-digits by 1-digit (sharing with exchange) |  |  |  |  |  | Year: 3/4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $52 \div 4=13$ | ? | ? | ? | ? | When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. <br> Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows. <br> Flexible partitioning in a part-whole model supports this method. |






| Skill: Divide 4-digits by 1-digit (grouping) |  |  |  |  |  | Year: 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $[8,532 \div 2=4,266]$ | 2 | 4 | 2 | 13 | 12 | Place value counters or plain counters can be used on a place value grid to support children to divide 4digits by 1 -digit. Children can also draw their own counters and group them through a more pictorial method. <br> Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges. |






## Glossary

Array - An ordered collection of counters, cubes or other item in rows and columns.

Commutative - Numbers can be multiplied in any order.

Dividend - In division, the number that is divided.

Divisor - In division, the number by which another is divided.

Exchange - Change a number or expression for another of an equal value.

Factor - A number that multiplies with another to make a product.

Multiplicand - In multiplication, a number to be multiplied by another.

Partitioning - Splitting a number into its component parts.

Product - The result of multiplying one number by another.

Quotient - The result of a division

Remainder - The amount left over after a division when the divisor is not a factor of the dividend.

Scaling - Enlarging or reducing a number by a given amount, called the scale factor

